Theoretical modelling of the laboratory negative stepped leader

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1. Introduction

Several theoretical studies have dealt with the self consistent simulation of the laboratory spark under positive polarity [1][2][3]. There is obviously a need for similar modelling of the negative discharge for different practical purposes: electrical engineering, lightning research, understanding of the aircraft striking process, etc...

This paper presents a new model for the simulation of the propagation of a negative stepped-leader over long air gaps. Previous laboratory experiments [4][5] have allowed to define the different subsequent phases of the stepped-leader process.

In the present work, specific models of these successive phases will be presented, based upon hypothesis on the fundamental physical mechanisms involved. These partial models are then linked into a complete model of the stepping process. An example of the simulation results will be presented: the calculated discharge parameters (stepping period, discharge size and velocity, current) will be compared to the experimental data available. The use of this model for the simulation of triggered or natural negative leaders will be discussed.

2. Experimental observations

The configuration studied here is basically a rod-plane gap subjected to negative switching surge voltage.

The streak cameragram of figure 1a shows that the negative discharge is a non-continuous phenomenon in which some abrupt elongation of the discharge channel, characterised by a strong reillumination ("step"), are separated by a complex sequence of phenomena ("interstep") which enable the discharge advancement.

The interstep processes can be subdivided into the following main phases:

- First corona development from the H.V. electrode (time t₁).
- After the first corona extinction, a short dark period takes place before the inception and development of a pilot system at the head of the discharge (time t₂). This system is made of two coronas of opposite polarity, the positive one develops towards the H.V. electrode, while the negative one propagates towards the grounded plane. At regular time intervals (time t₃, time t₄), new similar pilots develop from the tip of the preceding ones (see figure 1b).
- At time t₅, a negative leader starts from the negative electrode. At time t₆, a "spatial leader" develops from a "stem", actually a previous pilot inception point. These two leaders propagate in a convergent mode.
- At time t₇, the junction of the two leaders produces a strong reillumination of the whole channel, while a new corona starts at its tip. The entire process repeats starting from the tip of the new corona- until the discharge reaches the grounded plane.
Figure 1: Streak cameragram, associated current and simplified scheme of a negative discharge (2m rod-plane gap; 35/3000 μs waveshape; $V_C=1200$ KV).

3. Principles of modelling

3.1 Simulation of the first corona inception and development

The first corona is a set of branched filamentary channels (streamers) developing from the H.V. electrode and injecting into the gap a net negative charge. The main mechanisms for its inception and development are similar to those discussed in case of positive polarity [2]: 
Provided that the electric field strength attains sufficiently high values in an "active volume" around the H.V. electrode, the discharge starts when a suitably placed electron is generated. Then an electron avalanche greater than the critical size can develop by impact ionisation (the critical size is the minimum number of ions in the active head $N_{ct}$ for a stable reproduction of the streamer tip). The condition for inception is identical in both polarities and can be described by the following equation:

$$\exp\left\{\int_{\Delta z} (\alpha - \eta)dz\right\} \geq N_{ct}$$

(1)

where:
- $\alpha, \eta$: ionisation and attachment coefficient;
- $\Delta z$: length of the electron avalanche;

The use of this criteria allows the calculation of the first corona inception time.

![Figure 2 - Scheme of the active volume where a streamer initiatory avalanche can develop, in both positive and negative polarity.](image)

At the streamer tip, ions and excited molecules are highly concentrated, as result of previous ionisation and excitation effects (figure 3). The decay of the excited states produces, by photo ionisation, secondary electrons around the tip. If the field resulting from the combination of the local space charge and electrode potential are high enough, these free electrons drifting in the field directions, will form new avalanches which sustain the advancement of the discharge.

The overall process can be described by an energy balance giving the spatial or temporal evolution of the number of ions $N_s$ contained in the streamer active head, as function of the geometric potential $V(x)$ distribution [6], [7]:

$$N_s(x) = N_0 + \frac{R(e\Delta x + \mu)}{2a\Delta x} \left\{ V(0) - \frac{\gamma}{e\Delta x + \mu} x - V(x) \right\}$$

(2)
where:
- \( x \): linear coordinate along the propagation line;
- \( R \): radius of the streamer head;
- \( \Delta x \): individual propagation step (fig. 3);
- \( \mu, \gamma \): electron gain and loss coefficient in an avalanche formation;
- \( c \): electronic charge;
- \( N_0 \): ions number at \( x=0 \).

![Diagram](image)

**Figure 3** - Scheme of the avalanche development in both positive (a) and negative (b) streamer development.

In positive polarity, this equation provides the calculation of the final streamer length and charge [7].

In negative polarity the electrons move away from the streamer head leaving a positive charged zone which is neutralised by the previous negative head in a local micro discharge (figure 3b). Owing to the different mechanism leading to avalanche development, the physical parameters used in the positive streamer simulation have been recalculated: the resulting energy balance allows to calculate the final corona length.

The calculation of the net negative charge associated with the development of the negative corona is performed using the assumption that the electric field within the corona region assumes a constant value and that the corona space charge assumes that distribution and value which are necessary to rise the potential from the geometric distribution \( V(x) \) to that after the corona development \( V_f(x) \), (figure 4). It can be shown [8] that:

\[
Q = \frac{4 \pi \varepsilon_0}{\alpha L_x} \int_{0}^{L_i} (V_f(x) - V(x)) dx
\]  

(2)
with:
\[ Q \quad : \quad \text{net negative corona charge;} \]
\[ L_s \quad : \quad \text{streamer length;} \]
\[ \alpha \quad : \quad \text{coefficient function of corona geometry.} \]

![Graph showing potential distributions](image)

**Figure 4** - Potential distributions before (a) and after (b) corona development, during the relaxation process (c), and after the pilot formation (d).

### 3.2 Inception and development of a "pilot" system.

Each streamer filament within the corona is a plasma channel of limited conductivity: after extinction of the negative corona and of the associated current, this plasma undergoes a classical relaxation process which tends to increase the local potential towards the H.V. electrode potential, in order to reduce to zero the internal field. The relaxation time constant depends essentially on the corona non uniform geometry and charge distribution.

A non-uniform multiple resistor-capacitance line model has been retained to describe this relaxation phenomenon. Resistance values are defined upon local conductivity conditions derived from the corona model, which provides the number of ions \( N \) per unit length (eq. 2)

\[
\text{the linear capacitance has been considered constant.}
\]

The evolution of the corresponding potential profile is given in figure 4. It can be seen that the increasing potential at the streamer head may create the electric field needed for the inception of a new negative corona: in this case the new generated electrons and the local potential redistribution trigger a positive corona developing up to the H.V. electrode, provided the mean geometric field in this region is higher than the positive corona stability field.

The corona plasma relaxation and the inception at one point of its border of a bidirectional positive and negative corona system has been assumed as the "pilot" mechanism of the propagation of the negative discharge into the gap. At each time-step of the computation, the inception conditions for the negative and positive coronas are tested. When these conditions are fulfilled, the development of both coronas is simulated using the models described in §3.1. After the extinction of these newly formed coronas, another relaxation process takes place along the body of pilot system; it can be simulated using the same method, starting from the new
space charge and potential distributions (figure 4). The repetition of successive pilot systems at more and more advanced positions in the gap can therefore be modelled; the computed results are reported in figure 5: the pilot repetition frequency and average propagation velocity are in good agreement with experimental results (see figure 1). The current (figure 5b) has a pulsed component, essentially related to the positive and negative corona formation and a more or less continuous component essentially related to the plasma relaxation. Also the total charge values are in good agreement with the experimental results.

![Graph](image)

Figure 5 - Simulation of the streak photograph of the interstep phase in a negative discharge and of the associated current.

3.3 Inception and development of the electrode and space leader.

The formation of the positive or negative leader channel [2] is associated to the heating of a portion of the corona filaments (stem) up to a critical temperature of the order of 1500K: at this temperature, the detachment of negative ions causes an abrupt release of free electrons in the plasma, increasing strongly its conductivity. Under positive polarity the stems are formed on the H.V. electrode, at the root of the corona filaments; under negative polarity the stems are also observed in midgap in correspondence of the "seed" point of the pilot system. In any case the formation of conductive stem concentrates the field lines at its tip and may cause the development of a new corona, if the inception condition (1) is fulfilled.

In negative polarity, two type of leaders may be distinguished:
• As in positive polarity, a leader which develops from the H.V. electrode: in this case the stem is represented as a small cylindrical protrusion from the surface of the H.V. electrode.

• A spatial leader which start in the pilot system origin and develops in two opposite directions: in this case the stem is represented as a small conductive ellipsoid which modifies the shape of the local field lines.

The first corona and the subsequent pilot systems inject the current input necessary for the streamer to leader transition. An energy balance can be written to describe this process [2], taking into account the different energy reservoirs of the gas molecules; it has been demonstrated that the electrical energy is mainly transferred to vibrational levels whose relaxation time governs the kinetics of the overall heating process. The energy balance can be used to calculate the stem temperature and therefore as a test for leader inception at each time step of the computation.

After this thermodynamic condition is fulfilled, the criteria for inception of a new corona has to be satisfied, this new corona being the source for input current sustaining the leader propagation.

In the model for both electrode and space leaders the thermodynamic condition is similar but the corona inception condition is different owing to different local electric field conditions. The relaxation process giving rise to the pilot formation at the head of the discharge remains almost unchanged with the inception of the leaders (figure 5a). However, the presence of the highly conductive leader channels modify the current associated with the relaxation process and a continuous component is then superimposed to the pulses (figure 5b).

The advancement of the electrode and space leaders is governed by the current injected at each leader head by the corona and pilot systems; this current has been calculated respectively from the corona and R-C line relaxation models.

Under positive polarity, it has been demonstrated that the leader tip velocity \( v_L \) is related to the injected current \( I_L \) by the following equation:

\[
v_L = \frac{I_L}{q_L}
\]  

(3)

Complex expressions for the charge per unit length \( q_L \) can be derived [2]; in the actual model the physical and geometrical parameters in these expressions have been adapted to the case of the electrode negative leader and of the positive and negative head of the space leader. The following values have been obtained:

- negative head: \( q_L = 145 \, \mu\text{C} \)
- positive head: \( q_L = 73.5 \, \mu\text{C} \)

which are consistent with experimental measurements [9].

3.5 Junction process

The advancement of the positive upper leader head toward the H.V. electrode leads to a junction with the downward negative leader moving in the opposite direction, provided the injected currents are sufficient to allow a stable propagation all the leader heads.

Two types of junctions are distinguished: the connection between two space leaders (fig. 1c, time \( t_5 \)) and the connection of the space leader with the negative leader from the H.V. electrode (fig. 1c, time \( t_6 \)).

Experimental observations indicate that the junction between two space leaders has no strong influence on the overall discharge evolution, while the junction of a space leader with the
negative electrode leader causes a strong reillumination of all the connected channels, the end of interstep phase and the start of a new strong corona. Owing to the large current injected at this time, it is possible to assume that the resulting leader will be highly conductive due to thermal ionisation processes [8]. The potential profiles prior and after this type of junction are presented in figure 6. The abrupt potential variation causes a strong enhancement of field and potential at the leader tip and therefore the inception of a new large negative corona. In the model, the use of equation (2) allows to calculate the charge associated with the potential re-distribution. The start of a new interstep phase can then be simulated from that point using the methods described in the previous paragraphs. The final stop condition is obtained when the corona head reaches the opposite plane.

Figure 6 - Potential distribution before (a) and after (b) the junction of a negative and a space leader.

4. Results

The model has been used to simulate the negative discharge development in the case of a rod-plane configuration experimentally studied by the Renardières Group in 1978 [4]. The following parameters have been used in the computation:

- Gap length : $D=7$ meters.
- Electrode curvature radius $R_c = 30$ cm.
- Voltage waveform $6/3000 \mu s$ or $60/3000 \mu s$ with a crest voltage of 2.8 MVolts.

Figures 7b and 8b presents the results of the computation for the two front durations, in comparison with the experimental streak camerograms of figures 7a and 8a. The overall computed space-time characteristics of the discharge are in very good agreement with the experimental records. The average stepping period is found around 15 to 20 $\mu s$. The calculated current is made of series of pulses which are consistent with current measurements.

The best agreement between the model and the experimental data has been obtained after an optimisation of the following parameters: R-C line capacitance, spatial stem dimensions, approximations for analytical calculation of the electric field at the pilot starting point. Further experimental data would be useful to test the validity of these parameters in a wide set of configurations.
5. Conclusion

The proposed model attempts a physical description of the different phases of the stepped propagation of the negative discharge in long gaps. It has been successfully tested for laboratory discharge conditions; it will be tested in the near future for the negative lightning triggered in altitude. The model can be used in various ambient field profiles and provide quantitative informations concerning current shapes, discharge geometry and velocity. The final objective of this work will be to use this model, together with the similar one developed for the positive discharge [10], in order to provide a simulation of the bidirectional leader development which initiates the aircraft and launchers striking process.

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Figure 8 - Comparison between experimental and computed streak photographs.

References:

[9] Castellani A. et al. - "Laboratory simulation of the bleader process on an electrically floating conductor", in course of publication.